1. The functions f and g are such that

$$f(x) = 5x + 3 \qquad g(x) = ax + b \qquad \text{where } a \text{ and } b \text{ are constants.}$$

$g(3) = 20$ and $f^{-1}(33) = g(1)$

Find the value of a and the value of b.

$f(x) = 5x + 3$

$$f(x)$$
$$\begin{tikzcd} x \arrow[r, bend left, "\times 5"] & 5x \arrow[l, bend left, "\div 5"] \arrow[r, bend left, "+3"] & 5x+3 \arrow[l, bend left, "-3"] \end{tikzcd}$$
$$f^{-1}(x)$$

$f^{-1}(x) = \dfrac{x-3}{5}$

$f^{-1}(x) = \dfrac{(33)-3}{5}$

$= \dfrac{30}{5}$

$= 6$

$6 = g(1)$

$6 = a(1) + b$

$6 = a + b$

$g(3) = a(3) + b$

$= 3a + b$

$20 = 3a + b$

① $20 = 3a + b$

② $6 = a + b$

① - ②

$14 = 2a$

$(\div 2) \quad (\div 2)$

$7 = a$

$6 = a + b$

$6 = (7) + b$

$(-7) \quad (-7)$

$-1 = b$

$a = $ 7

$b = $ -1

(Total for Question is 5 marks)

2. f and g are functions such that

$$f(x) = \frac{2}{x^2} \quad \text{and} \quad g(x) = 4x^3$$

(a) Find f(–5)

Substitute $x = -5$ into $f(x)$ function.

$$f(-5) = \frac{2}{(-5)^2} = \frac{2}{25}$$

$\frac{2}{25}$

(1)

(b) Find fg(1)

composite function

'do g then do f' $= f(g(x)) = f(g(1))$

$g(1) = 4 \times 1^3$ 1

$= 4$

$f(4) = \frac{2}{4^2} = \frac{2}{16} = \frac{1}{8}$

$\frac{1}{8}$ 1

(2)

(Total for Question is 3 marks)

3. For all values of x

$$f(x) = (x + 1)^2 \qquad \text{and} \qquad g(x) = 2(x - 1)$$

(a) Show that $gf(x) = 2x(x + 2)$

$gf(x) = g(f(x))$

$f(x) = (x+1)^2$

$g(x) = ((x+1)^2) = 2((x+1)^2 - 1)$

$= 2(x^2 + 2x + 1 - 1)$

$= 2(x^2 + 2x)$

$= 2x(x+2)$

(2)

(b) Find $g^{-1}(7)$

$g(x) = 2(x-1)$

$= 2x - 2$

$$\begin{tikzcd} x \arrow[r, bend left, "\times 2"] & 2x \arrow[l, bend left, "\div 2"] \arrow[r, bend left, "-2"] & 2x-2 \arrow[l, bend left, "+2"] \end{tikzcd}$$

$g^{-1}(x) = \dfrac{x+2}{2}$

$g^{-1}(7) = \dfrac{(7)+2}{2}$

$= \dfrac{9}{2}$

$\dfrac{9}{2}$

(2)

4. The functions f and g are such that

$$f(x) = 3x - 1 \quad \text{and} \quad g(x) = x^2 + 4$$

(a) Find $f^{-1}(x)$

$f(x) = 3x - 1.$

$y = 3x - 1.$ (1)

$y + 1 = 3x.$

$\div 3$ $\frac{y+1}{3} = x$ $\div 3$

$x = \frac{y+1}{3}$

$\boxed{\therefore f^{-1}(x) = \frac{x+1}{3}}$

(1) $f^{-1}(x) = \frac{x+1}{3}$

(2)

Given that $fg(x) = 2gf(x)$,

(b) show that $15x^2 - 12x - 1 = 0$

$f(x) = 3x - 1. \quad g(x) = x^2 + 4.$

Find $fg(x)$:

$fg(x) = f(g(x)) = f(x^2 + 4).$

$f(x^2+4) = 3(x^2+4) - 1 = 3x^2 + 12 - 1 = 3x^2 + 11.$

$\rightarrow fg(x) = 3x^2 + 11.$ (1)

Find $gf(x)$:

$gf(x) = g(f(x)) = g(3x-1).$

$g(3x-1) = (3x-1)^2 + 4 = (9x^2 - 6x + 1) + 4 = 9x^2 - 6x + 5.$ (1)

$\rightarrow gf(x) = 9x^2 - 6x + 5.$ (1)

$fg(x) = 2gf(x).$ (1)

$3x^2 + 11 = 2(9x^2 - 6x + 5).$

$3x^2 + 11 = 18x^2 - 12x + 10.$

$0 = 15x^2 - 12x - 1.$

(1) $\boxed{\therefore 15x^2 - 12x - 1 = 0}$

(5)

(Total for Question is 7 marks)

5. The function f is given by

$$f(x) = 2x^3 - 4$$

(a) Show that $f^{-1}(50) = 3$

$x = 2y^3 - 4$

$x + 4 = 2y^3$

$y^3 = \frac{x+4}{2}$

$y = \sqrt[3]{\frac{x+4}{2}}$

$f^{-1}(x) = \sqrt[3]{\frac{x+4}{2}}$ 1

$\therefore f^{-1}(50) = \sqrt[3]{\frac{50+4}{2}} = \sqrt[3]{\frac{54}{2}} = \sqrt[3]{27} = 3$ 1

(2)

The functions g and h are given by

$$g(x) = x + 2 \quad \text{and} \quad h(x) = x^2$$

(b) Find the values of x for which

$$hg(x) = 3x^2 + x - 1$$

$h(g(x)) = h(x+2) = (x+2)^2$

$\therefore hg(x) = (x+2)^2$ 1

$(x+2)^2 = 3x^2 + x - 1$

$(x+2)(x+2)$ 1

$x^2 + 4x + 4 = 3x^2 + x - 1$

$4x + 4 = 2x^2 + x - 1$

$4 = 2x^2 - 3x - 1$

$0 = 2x^2 - 3x - 5$

$2x^2 - 3x - 5 = 0$ 1

$(2x - 5)(x + 1) = 0$

$2x - 5 = 0$ $\quad$ $x + 1 = 0$

$x = 5/2$ $\quad$ $x = -1$

1

$x = 5/2$ and $x = -1$

(4)

6. f and g are functions such that

$$f(x) = \frac{12}{\sqrt{x}} \quad \text{and} \quad g(x) = 3(2x + 1)$$

(a) Find g(5)

↳ Substitute 5 for x in g

$g(5) = 3(2\times5+1)$

$= 3(11) = 33$

33 ✓1

(1)

(b) Find gf(9)

$f(x) = \frac{12}{\sqrt{x}}$ $\quad g(x) = 3(2x+1)$

$g(f(9))$

$f(9) = \frac{12}{\sqrt{9}} = \frac{12}{3} = 4$ ✓1

$g(f(9)) = g(4) = 3(2\times4+1)$

$= 27$

27 ✓2

(2)

(c) Find $g^{-1}(6)$

$g(x) = 3(2x+1)$

① finding $g^{-1}(x)$

$\rightarrow x = 3(2y+1)$ (rearrange for y)

$\div 3$

$\frac{x}{3} = 2y+1$

-1

$\frac{x}{3} - 1 = 2y$

$\div 2$

$y = \frac{1}{2}\left(\frac{x}{3} - 1\right)$

$\therefore g^{-1}(x) = \frac{1}{2}\left(\frac{x}{3} - 1\right)$ ✓1

$g^{-1}(6) = \frac{1}{2}\left(\frac{6}{3} - 1\right)$

$= \frac{1}{2} \times 1 = \frac{1}{2}$

1/2 ✓2

(2)

(Total for Question is 5 marks)

7. The functions f and g are such that

$$f(x) = 3x^2 + 1 \quad \text{for } x > 0 \qquad \text{and} \qquad g(x) = \frac{4}{x^2} \quad \text{for } x > 0$$

(a) Work out gf(1) $g(f(1))$.

Start with f(1) : $f(1) = 3(1)^2 + 1 = 3 + 1 = 4.$ (1)

$f(1) = 4 \therefore g(f(1)) = g(4) = \frac{4}{4^2} = \frac{4}{16} = \frac{1}{4}$

(1) $\frac{1}{4}$

(2)

The function h is such that $h = (fg)^{-1}$

(b) Find h(x)

$(fg)^{-1}$ is the inverse of $f(g)$.

Find f(g) : $f(x) = 3x^2 + 1.$

$$f\left(\frac{4}{x^2}\right) = 3\left(\frac{4}{x^2}\right)^2 + 1.$$

(1)

$$= 3\left(\frac{16}{x^4}\right) + 1 = \frac{48}{x^4} + 1.$$

$f(g) = \frac{48}{x^4} + 1.$ Find inverse :

Let $y = \frac{48}{x^4} + 1.$ Make x the subject

(1)

$$y - 1 = \frac{48}{x^4}.$$

$$\times x^4 \qquad x^4(y-1) = 48. \qquad \times x^4$$

$$\div(y-1) \qquad x^4 = \frac{48}{y-1} \qquad \div(y-1)$$

(1)

$$x = \sqrt[4]{\frac{48}{y-1}}$$

Now swap the y with an x, and swap the x with $(fg)^{-1}$:

$$\boxed{(fg)^{-1} = \sqrt[4]{\frac{48}{x-1}}}$$

(1) $\sqrt[4]{\frac{48}{x-1}}$

(4)

(Total for Question is 6 marks)