

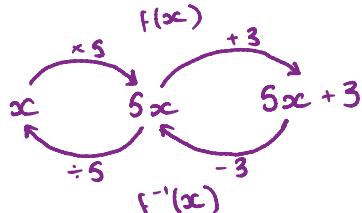
1. The functions f and g are such that

$$f(x) = 5x + 3 \quad g(x) = ax + b \quad \text{where } a \text{ and } b \text{ are constants.}$$

$$g(3) = 20 \quad \text{and} \quad f^{-1}(33) = g(1)$$

Find the value of a and the value of b .

$$f(x) = 5x + 3$$



$$6 = g(1)$$

$$6 = a(1) + b$$

$$6 = a + b \quad /$$

$$f^{-1}(x) = \frac{x-3}{5} \quad /$$

$$f^{-1}(x) = \frac{(33)-3}{5}$$

$$= \frac{30}{5}$$

$$= 6 \quad /$$

$$\begin{aligned} g(3) &= a(3) + b \\ &= 3a + b \end{aligned}$$

$$20 = 3a + b$$

$$\textcircled{1} \quad 20 = 3a + b$$

$$\textcircled{2} \quad 6 = a + b$$

$$\textcircled{1} - \textcircled{2}$$

$$14 = 2a$$

$$(\div 2) \quad (\div 2)$$

$$7 = a$$

$$6 = a + b$$

$$6 = (7) + b$$

$$(-7) \quad (-7)$$

$$-1 = b$$

$$a = \dots \quad 7 \quad /$$

$$b = \dots \quad -1 \quad /$$

(Total for Question is 5 marks)

2. f and g are functions such that

$$f(x) = \frac{2}{x^2} \quad \text{and} \quad g(x) = 4x^3$$

(a) Find $f(-5)$ *Substitute $x = -5$ into $f(x)$ function.*

$$f(-5) = \frac{2}{(-5)^2} = \frac{2}{25}$$

..... (1)

- (b) Find $fg(1)$

Composite function

'do g then do f' = $f(g(x)) = f(g(1))$

$$g(1) = 4 \times 1^3 \textcircled{1}$$

$$= 4$$

$$f(4) = \frac{2}{4^2} = \frac{2}{16} = \frac{1}{8}$$

..... (2)

(Total for Question is 3 marks)

3. For all values of x

$$f(x) = (x + 1)^2 \quad \text{and} \quad g(x) = 2(x - 1)$$

- (a) Show that $gf(x) = 2x(x + 2)$

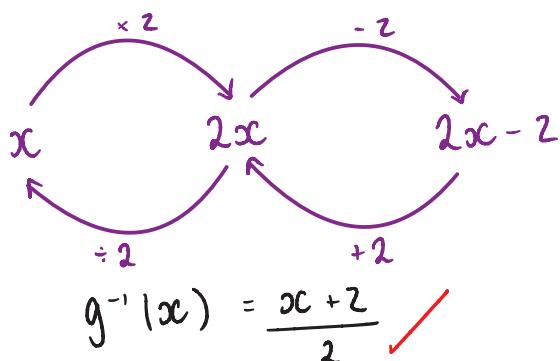
$$gf(x) = g(f(x))$$

$$\begin{aligned} f(x) &= (x + 1)^2 \\ g(x) &= ((x + 1)^2) = 2((x + 1)^2 - 1) \\ &= 2(x^2 + 2x + 1 - 1) \\ &= 2(x^2 + 2x) \\ &= 2x(x + 2) \end{aligned}$$

(2)

- (b) Find $g^{-1}(7)$

$$\begin{aligned} g(x) &= 2(x - 1) \\ &= 2x - 2 \end{aligned}$$



$$\begin{aligned} g^{-1}(7) &= \frac{(7)+2}{2} \\ &= \frac{9}{2} \end{aligned}$$

$\frac{9}{2}$ ✓

(2)

4. The functions f and g are such that

$$f(x) = 3x - 1 \quad \text{and} \quad g(x) = x^2 + 4$$

(a) Find $f^{-1}(x)$

$$\begin{aligned} f(x) &= 3x - 1 & x &= \frac{y+1}{3} \\ y &= 3x - 1 & \text{①} \\ y+1 &= 3x \\ \div 3 & \left(\begin{array}{l} y+1 = 3x \\ \frac{y+1}{3} = x \end{array} \right) \div 3 & \text{②} \\ & \therefore f^{-1}(x) = \frac{x+1}{3} \\ & f^{-1}(x) = \frac{x+1}{3} & \text{③} \end{aligned}$$

Given that $fg(x) = 2gf(x)$,

(b) show that $15x^2 - 12x - 1 = 0$

$$f(x) = 3x - 1, \quad g(x) = x^2 + 4.$$

Find $fg(x)$:

$$\begin{aligned} fg(x) &= f(g(x)) = f(x^2 + 4) \\ f(x^2 + 4) &= 3(x^2 + 4) - 1 = 3x^2 + 12 - 1 = 3x^2 + 11. \\ \rightarrow fg(x) &= 3x^2 + 11. \quad \text{④} \end{aligned}$$

Find $gf(x)$:

$$\begin{aligned} gf(x) &= g(f(x)) = g(3x - 1). \\ g(3x - 1) &= (3x - 1)^2 + 4 = (9x^2 - 6x + 1) + 4 = 9x^2 - 6x + 5. \\ \rightarrow gf(x) &= 9x^2 - 6x + 5. \quad \text{⑤} \end{aligned}$$

$$fg(x) = 2gf(x). \quad \text{⑥}$$

$$3x^2 + 11 = 2(9x^2 - 6x + 5).$$

$$\underbrace{3x^2 + 11}_{\text{⑦}} = 18x^2 - 12x + 10.$$

$$0 = 15x^2 - 12x - 1. \quad \text{⑧}$$

(Total for Question is 7 marks)

5. The function f is given by

$$f(x) = 2x^3 - 4$$

- (a) Show that $f^{-1}(50) = 3$

$$\begin{aligned} x &= 2y^3 - 4 & f^{-1}(x) &= \sqrt[3]{\frac{x+4}{2}} \quad (1) \\ x+4 &= 2y^3 \\ y^3 &= \frac{x+4}{2} \\ y &= \sqrt[3]{\frac{x+4}{2}} \end{aligned}$$

∴ $f^{-1}(50) = \sqrt[3]{\frac{50+4}{2}} = \sqrt[3]{\frac{54}{2}} = \sqrt[3]{27} = 3 \quad (1)$

(2)

The functions g and h are given by

$$g(x) = x + 2 \text{ and } h(x) = x^2$$

- (b) Find the values of x for which

$$hg(x) = 3x^2 + x - 1$$

$$h(g(x)) = h(x+2) = (x+2)^2$$

$$\therefore hg(x) = (x+2)^2 \quad (1)$$

$$(x+2)^2 = 3x^2 + x - 1$$

$$\downarrow (x+2)(x+2) \quad (1)$$

$$x^2 + 4x + 4 = 3x^2 + x - 1$$

$$4x + 4 = 2x^2 + x - 1$$

$$4 = 2x^2 - 3x - 1$$

$$0 = 2x^2 - 3x - 5$$

$$\checkmark 2x^2 - 3x - 5 = 0 \quad (1)$$

$$(2x - 5)(x + 1) = 0$$

$$\left. \begin{array}{l} 2x - 5 = 0 \\ x = \frac{5}{2} \end{array} \right.$$

$$\left. \begin{array}{l} x + 1 = 0 \\ x = -1 \end{array} \right.$$

(1)

$$x = \frac{5}{2} \text{ and } x = -1$$

(4)

6. f and g are functions such that

$$f(x) = \frac{12}{\sqrt{x}} \quad \text{and} \quad g(x) = 3(2x + 1)$$

- (a) Find $g(5)$

↪ Substitute 5 for x in g

$$\begin{aligned} g(5) &= 3(2 \times 5 + 1) \\ &= 3(11) = 33 \end{aligned}$$

33 ✓

(1)

- (b) Find $gf(9)$

$$f(x) = \frac{12}{\sqrt{x}} \quad g(x) = 3(2x + 1)$$

$g(f(9))$

$$f(9) = \frac{12}{\sqrt{9}} = \frac{12}{3} = 4 \quad \checkmark$$

$$\begin{aligned} g(f(9)) &= g(4) = 3(2 \times 4 + 1) \\ &= 27 \end{aligned}$$

27 ✓

(2)

- (c) Find $g^{-1}(6)$

$$g(x) = 3(2x + 1)$$

① finding $g^{-1}(x)$

$$\begin{aligned} &\rightarrow x = 3(2y + 1) \quad (\text{rearrange for } y) \\ &\div 3 \quad \frac{x}{3} = 2y + 1 \quad \downarrow \div 3 \\ &-1 \quad \frac{x}{3} - 1 = 2y \quad \downarrow -1 \\ &\div 2 \quad y = \frac{1}{2} \left(\frac{x}{3} - 1 \right) \quad \downarrow \div 2 \\ &\therefore g^{-1}(x) = \frac{1}{2} \left(\frac{x}{3} - 1 \right) \quad \checkmark \end{aligned}$$

(Total for Question is 5 marks)

7. The functions f and g are such that

$$f(x) = 3x^2 + 1 \quad \text{for } x > 0 \quad \text{and} \quad g(x) = \frac{4}{x^2} \quad \text{for } x > 0$$

(a) Work out $gf(1)$.

Start with $f(1)$: $f(1) = 3(1)^2 + 1 = 3 + 1 = 4.$ (1)

$$f(1) = 4 \therefore g(f(1)) = g(4) = \frac{4}{4^2} = \frac{4}{16} = \frac{1}{4}$$

(1) $\frac{1}{4}$

(2)

The function h is such that $h = (fg)^{-1}$

(b) Find $h(x)$

$(fg)^{-1}$ is the inverse of $f(g).$

Find $f(g)$: $f(x) = 3x^2 + 1.$

$$\begin{aligned} f\left(\frac{4}{x^2}\right) &= 3\left(\frac{4}{x^2}\right)^2 + 1. \\ &= 3\left(\frac{16}{x^4}\right) + 1 = \frac{48}{x^4} + 1. \end{aligned}$$

(1)

$f(g) = \frac{48}{x^4} + 1.$ Find inverse:

Let $y = \frac{48}{x^4} + 1.$ Make x the subject

(1)

$$y - 1 = \frac{48}{x^4}.$$

(1) $\sqrt[4]{\frac{48}{x-1}}$

(4)

x^4 (y-1) \downarrow x^4 \downarrow

$$x^4(y-1) = 48.$$

(Total for Question is 6 marks)

$\div(y-1)$ \downarrow $x^4 = \frac{48}{y-1}$ $\downarrow \div(y-1)$

NOW swap the y with an x , and swap the x with $(fg)^{-1}:$

(1)

$$x = \sqrt[4]{\frac{48}{y-1}}$$

$$(fg)^{-1} = \sqrt[4]{\frac{48}{x-1}}$$